Pregroups and distributional semantics

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Meaning as a distribution?

The distributional hypothesis

You shall know a word by the company it keeps. (Firth)

Words that occur in the same contexts tend to have similar meanings.

Requirements

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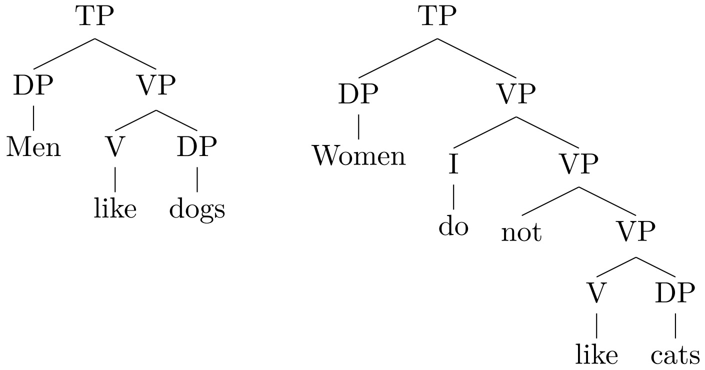

Lambek pregroups

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In both cases, the reduction rules are formalized through a partial order.

Axioms of a pregroup

Definition

A pregroup is a monoid with a partial order $\leq$ and two unary operations ^{l} and ^{r} such that :

- if $a \leq b$ then $ca \leq cb$ and $ac \leq bc$ for all c
- $x^l x \leq 1 \leq x x^l$ and $x x^r \leq 1 \leq x^r x$

	Calculus	**Pregroup**
(a)	$z \to z$	reflexivity of $\leq$
(b)	$(xy)z \to x(yz)$	associativity of $\cdot$
(c)	if $xy \to z$, then $x \to z/y$	$x = x1 \leq xyy^l = zy^l$
(d)	if $x \to z/y$, then $xy \to z$	$xy = zy^l y \leq z1 = z$
(e)	if $x \to y$ and $y \to z$ then $x \to z$	transitivity of $\leq$

Simplified grammar of English : n for nouns, s for sentences, j for infinitive verbs.

men	like	cute	dogs
n	$n^r s n^l$	$n n^l$	n

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Definition

A **monoidal category** is a category with a monoid law $\otimes$ on the objects, such that for each pair of arrows ($f : A \to B$, $g : C \to D$) there exists $f \otimes g : A \otimes C \to B \otimes D$ and such that

$$(f_1 \otimes g_1) \circ (f_2 \otimes g_2) = (f_1 \circ f_2) \otimes (g_1 \circ g_2)$$

In a set theoretic interpretation : $f \otimes g$ is just the parallel evaluation of f and g.

Definition

A **compact closed category** is a monoidal category such that for each object A, there exists :

- A^l and A^r
- $\eta^l : I \to A \otimes A^l$
- $\eta^r : I \to A^r \otimes A$
- $\epsilon^l : A^l \otimes A \to I$
- $\epsilon^r : A \otimes A^r \to I$

and the *yanking equalities* are satisfied.

The yanking equalities

The following equations are called *yanking equalities* :

$$(1_A \otimes \epsilon^l) \circ (\eta^l \otimes 1_A) = 1_A$$

$$(\epsilon^r \otimes 1_A) \circ (1_A \otimes \eta^r) = 1_A$$

$$(\epsilon^l \otimes 1_{A^l}) \circ (1_{A^l} \otimes \eta^l) = 1_{A^l}$$

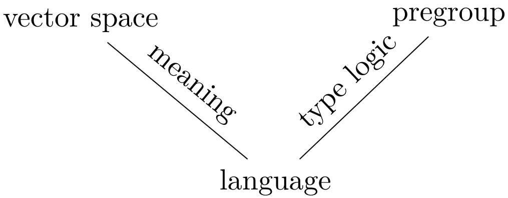
$$(1_{A^r} \otimes \epsilon^r) \circ (\eta^r \otimes 1_{A^r}) = 1_{A^r}$$

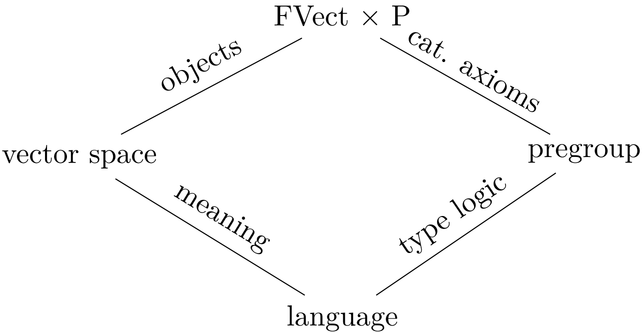
Vector spaces

The category of vector spaces is a compact closed category :

- for the monoid law $\otimes$ (Kronecker product) ;
- with $A^l = A^r = A^*$ (dual) ;
- with $\epsilon^l, \epsilon^r, \eta^l, \eta^r$ correctly defined.

Semantics

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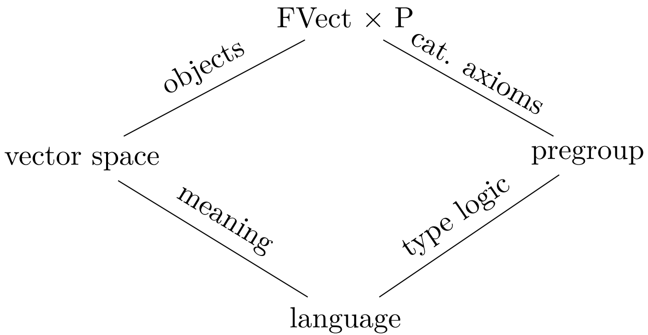
Semantics



men	like	dogs
$(\overrightarrow{men} \in W, n)$	$(\overrightarrow{like} \in W \otimes S \otimes W, n^r s n^l)$	$(\overrightarrow{dogs} \in W, n)$

Morphism associated to a reduction

If $a \leq b$ holds, then it can be deduced from the axioms :

$$x^l x \leq 1 \leq x x^l$$

$$x x^r \leq 1 \leq x^r x$$

Similarly, in vector spaces category described before, all the morphisms can be expressed using ϵ^l, ϵ^r, η^l and η^r (and 1_A, of course).

When $a \leq b$, we denote by $[a \leq b]$ the morphism of vector spaces associated to this reduction.

Meaning of a sentence

The meaning of the sentence $(v_1 \in W_1, p_1) \otimes \ldots \otimes (v_n \in W_n, p_n)$ is

$$[p_1 \cdot \ldots \cdot p_n \leq s](W_1 \otimes \ldots \otimes W_n)$$

.

$S = \mathbb{R}, \; W = \mathbb{R}^4$
Basis vectors for W : $\overrightarrow{female}$, $\overrightarrow{africa}$, $\overrightarrow{dog}$ and $\overrightarrow{idea}$.

$$\overrightarrow{women} \qquad \overrightarrow{dogs} \qquad \overrightarrow{like}$$

$$\begin{pmatrix} 0.8 & 0.05 & 0.05 & 0.1 \end{pmatrix} \quad \begin{pmatrix} 0.1 & 0 & 0.9 & 0 \end{pmatrix} \quad \begin{pmatrix} 0.1 & 0.2 & 0.6 & 0.1 \\ 0.25 & 0.5 & 0 & 0.25 \\ 0.4 & 0 & 0.6 & 0 \\ 0 & 0.5 & 0 & 0.5 \end{pmatrix}$$

An example (cont.)

Type reduction : $n\ n^r s n^l\ n \leq s$.
Morphism : $[nn^r sn^l n \leq s] = f : W \otimes W \otimes S \otimes W \otimes W \to S$.

$$f = \epsilon^r_W \otimes 1_S \otimes \epsilon^l_W$$

Meaning :

$$(\epsilon^r_W \otimes 1_S \otimes \epsilon^l_W)(\overrightarrow{women} \otimes \overrightarrow{like} \otimes \overrightarrow{dogs})$$

$$= (\epsilon^r_W \otimes 1_S \otimes \epsilon^l_W)(\sum_{i,j} c_{ij} \overrightarrow{women} \otimes \overrightarrow{u}_i \otimes \overrightarrow{v}_j \otimes \overrightarrow{dogs})$$

$$= \sum_{i,j} c_{ij} \langle \overrightarrow{women} | \overrightarrow{u}_i \rangle \langle \overrightarrow{u}_j | \overrightarrow{dogs} \rangle$$

$$= 0.47025$$

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